Notes 2/10 - Function Basics

Relation - a set of ordered pairs $(\overset{D}{x}, \overset{R}{y})$

FUNCTION - a special type of relation where every element in the $\overset{(x)}{\text{domain}}$ is paired exactly one element in the $\overset{(y)}{\text{range}}$

- x cannot repeat / go to more than one y

Domain: x-values (independent)

Range: y-values (dependent)

Ex 1: $\{ (2,5), (3,-2), (4,1), (2,3) \}$

{ } "set"

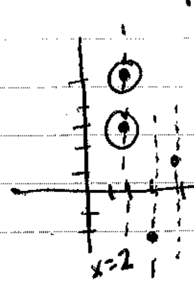
D: {2, 3, 4} R: {−2, 1, 3, 5}

* Write each # only once
* Write in order from least to greatest

Tests to decide if a relation is a function

1) Vertical line test

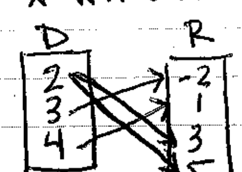
- plot points / graph ~~....~~
- If any vertical line hits the graph more than once ⇒ NOT a function



NOT a function

2) Mapping Diagram

- Create a box with #s from domain + a box with #s from the range
- Draw arrows to connect each x with its y from pt. (x, y)



NOT a function

- If any # in D has more than one arrow coming from it, NOT a function

Function Notation

"f of x" → $f(x) =$ means the same as $y =$

$v(t)$

__Evaluate__ ⇒ plug in the number or expression in ( ) wherever you see x

$f(x) = 2x^3 + 3x - 1$

Find $f(2)$ → replace all my x's w/ 2

$f(2) = 2(2)^3 + 3(2) - 1$

$f(2) = 2 \cdot 8 + 6 - 1 = 16 + 6 - 1 = 21$

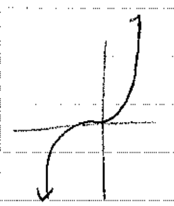
$f(2) = 21$ ⇒ means when $x = 2$, $y = 21$

$(2, 21)$ is on graph

Find $f(3z)$

$f(x) = 2x^3 + 3x - 1$

$f(3z) = 2(3z)^3 + 3(3z) - 1$



$= 2 \cdot 27z^3 + 9z - 1 = \underline{54z^3 + 9z - 1}$

Read

- Put in notes
- Try practice problems
- ✓ in
- Repeat × 3

Notes 2/13 - Composition of Functions

↳ put a function inside another function

Part 1: When the function is a finite set of points

limited #, not infinite

Ex: $f(x) = \{(7,8), (5,3), (9,8), (11,4)\}$

$g(x) = \{(5,7), (3,5), (7,9), (9,11)\}$

$f \circ g$

"f of g"

$f[g(x)]$

$\boxed{f \circ g = \{(5,8), (3,3), (7,8), (9,4)\}}$

Put g inside f

inputs of g — output of g

$f[g(5)] = f(7) = 8$

$f[g(3)] = f(5) = 3$

$f[g(7)] = f(9) = 8$

$f[g(9)] = f(11) = 4$

$g \circ f$

"g of f"

$g[f(x)]$

Put f inside g

$g[f(7)] = g(8) = DNE$

$g[f(5)] = g(3) = 5$

$g[f(9)] = g(8) = DNE$

$g[f(11)] = g(4) = DNE$

$\boxed{g \circ f = \{(5,5)\}}$

Notes 2/15 Composition of Functions Cont'd

Part 2: Composition with Evaluating

- Evaluate inside function first
- Take your number (answer, output) and plug it into the outside function

Ex 1:

$z(x) = 2x - 1 \qquad c(x) = x^2 + 1$

Find $z[c(-1)]$

Find $c(-1)$ first

$c = \{(-1, 2)$

$z = \{(2, 3)$

$c(-1) = (-1)^2 + 1 = 1 + 1 = \underline{2}$

$z(2) = 2(2) - 1 = 4 - 1 = \underline{3}$

$$\boxed{z[c(-1)] = 3}$$

Ex 2: Find $c[z(3)]$

Find $z(3)$ first!

$z(3) = 2(3) - 1 = 6 - 1 = 5$

$c(5) = 5^2 + 1 = 25 + 1 = \underline{26}$

$$\boxed{c[z(+3)] = 26}$$

practice

$g(x) = 5x \quad f(x) = x^2 \quad h(x) = x+4$

14) $f[g(1)]$

$g(1) = 5(1) = 5$

$f(5) = 5^2 = \boxed{25}$

15) $g[h(-2)] =$

$h(-2) = -2+4 = 2$

$g(2) = 5(2) = \boxed{10}$

16) $h[f(4)]$

$f(4) = 4^2 = 16$

$h(16) = \underset{x}{16} \underset{+4}{+4} = \boxed{20}$

18) $h[g(-3)]$

$g(-3) = 5(-3) = -15$

$h(-15) = -15+4 = \boxed{-11}$

21) $[\underset{3}{f} \circ (\underset{2}{h} \circ \underset{1}{g})](\overset{(x)}{-1}) = f[h(g(-1))]$

means plug in $x=-1$

1. $g(-1) = 5(-1) = -5$
2. $h(-5) = -5+4 = -1$
3. $f(-1) = (-1)^2 = \boxed{1}$

Part 3: Composition with Expressions

$f(x) = 3x \qquad g(x) = x^2 + x - 4$

$f \circ g = f[g(x)] = 3(g(x)) = 3(x^2 + x - 4) = \boxed{3x^2 + 3x - 12}$

↳ plug g(x) into f(x) wherever you see "x"

$g \circ f = g[f(x)] = (f(x))^2 + f(x) - 4$

$= (3x)^2 + 3x - 4$

$= \boxed{9x^2 + 3x - 4}$

Ex 2: $f(x) = 4x \qquad g(x) = 2x - 1$

$f \circ g = f[g(x)] = 4(g(x)) = 4(2x - 1) = \boxed{8x - 4}$

$g \circ f = g[f(x)] = 2(f(x)) - 1 = 2(4x) - 1$

$= \boxed{8x - 1}$

Practice 7-7

9) $g \circ h = g[h(x)] = -8(2x + 3) = \boxed{-16x - 24}$

$h \circ g = h[g(x)] = 2(-8x) + 3 = \boxed{-16x + 3}$

10) $g \circ h = g[h(x)] = \boxed{3x^2 + 6}$

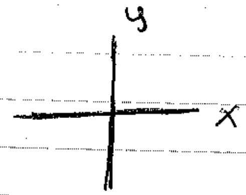
$h \circ g = h[g(x)] = 3(x + 6)^2$

$3(x^2 + 12x + 36) = \boxed{3x^2 + 36x + 108}$

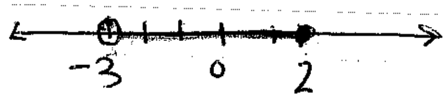
Notes 2/16 - Domain + Range on Graphs

D: X values (left, right)

R: y-values (down, up)



Interval notation

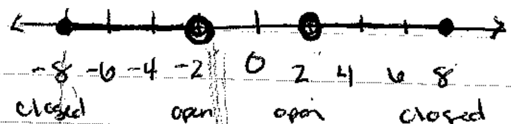


$= (-3, 2]$

↳ all #s between -3 and 2 not including 3, but including 2

open circle	closed circle
do NOT include #	do include #
use ( )	use [ ]

Ex 2:

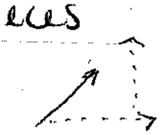


(lowest #, highest #)

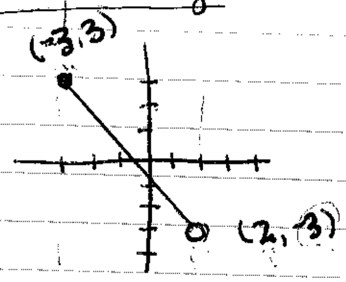
$[-8, -2) \cup (2, 8]$

↳ means "union" ⇒ want both pieces

Find domain & Range



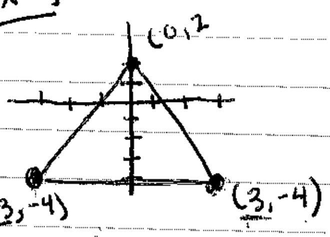
Ex 3:



D: $[-3, 2)$
left right

R: $(-3, 3]$
lowest highest

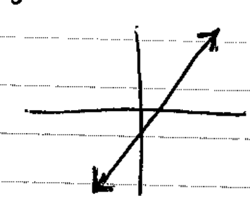
Ex 4:



D: $[-3, 3]$

R: $[-4, 2]$

Ex 5:



D: $(-\infty, \infty)$
infinity

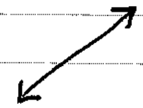
R: $(-\infty, \infty)$

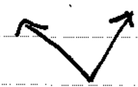
"All real numbers"

$\mathbb{R}$

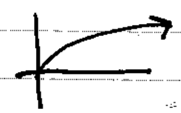
Summary of Function Families -Notes 2/29

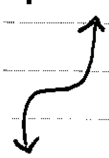
Quiz - Matching : name, equation, graph

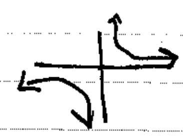
Linear $f(x) = x$ 

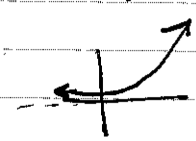
Absolute Value $f(x) = |x|$ 

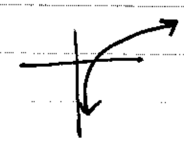
Quadratics $f(x) = x^2$ 

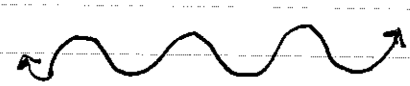
Square Root $f(x) = \sqrt{x}$ 

Cubic $f(x) = x^3$ 

Inverse $f(x) = \frac{1}{x}$ 

Exponential $f(x) = e^x$ 

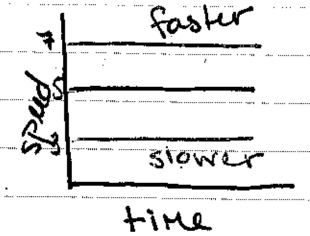
Logarithmic $f(x) = \ln(x)$ 

Sine $f(x) = \sin x$ 

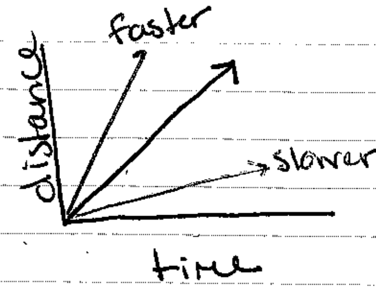
Trigonometric

Notes 3/1 - Graphs Tell a Story

Constant Speed

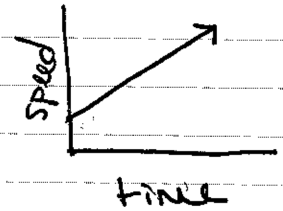


y-value = speed

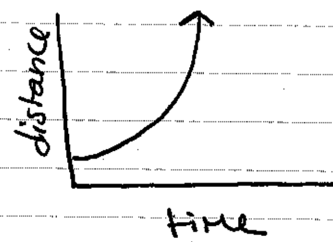


speed = slope

Accelerate
- increase speed

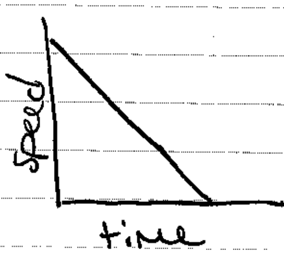


line w/ positive slope

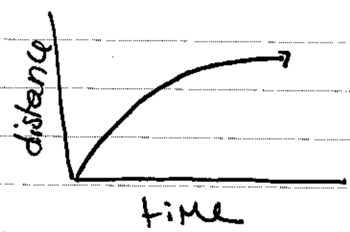


curve
concave up

Decelerate
- decrease speed



line with negative slope



curve
concave down

OR